This is the readme file for the R script “Sankhya\_code”, which contains codes relate to the paper with title “On the Distribution of Weighted Sum of Two Chi-squares with Applications to Shape Analysis”.

The codes have been divided into 6 chunks with indices ranged from 0 to 5.

**Code chunk 0** contains the codes for estimating the density function of the random variable AS, which is the weighed sum of two chi-square distributions defined in equation (7.5) in Section 7. The “convpow” function is used to obtain the estimates. The codes are wrapped up into a function with name “WS\_pdf”.

**Code chunk 1** contains the codes for reading in the data, converting them to arrays and computing the sample mean shapes. The data are first read in and saved as variables “Cle\_fir” and “Con\_fir”. Then they are converted into two arrays, “cle\_fir\_array” which involves 13 cleft subjects at the first frame, and “con\_fir\_array” which contains 12 control subjects at the first frame. At last, sample mean shapes for the cleft and control groups are computed separately and the results are saved in “Cle\_fir\_mean” and “Con\_fir\_mean”.

**Code chunk 2** contains the codes for computing the AS measure defined in equation (7.2) in Section 7. The results are saved with the names:

1. cle\_fir\_L2
2. con\_fir\_L2

for cleft and control subjects respectively.

**Code chunk 3** involves the codes pertained to Section 7.2.1, “Inference under Matrix Normality”. This code chunk is further divided into several sub-chunks:

1. In chunk 3.1, we first estimate the variance sigma^2 of the isotropic normal distribution given in equation (7.1) in Section 7. The estimation procedures are given in Section 6.1.1, where we first subtract the sample mean shape from each observation and vectorize each resulted matrix, then stack all these vectors together to a long vector and compute the sample variance. The results are saved with names “sig2\_clefir” and “sig2\_confir”.
2. In chunk 3.2, we compute the MoM estimates given in equation (6.1).
3. In chunk 3.3, we compute the hybrid moment estimates given in equation (6.4) based on equation (7.6). We first compute the asymmetry measure for the sample mean shape, then divide by the corresponding estimate of sigma^2. The resulted estimates for lambda1 are saved under the names “lambda1\_clefir” and “lambda1\_confir” for the cleft and control groups respectively.
4. In chunk 3.4, we compute the expectation and variance of the random variable AS using equation (7.11) for the two groups separately.
5. In chunk 3.5, the density function of the random variable AS for cleft and control groups are plotted on the same figure.
6. In chunk 3.6, we compute the direct measure of asymmetry defined in equation (7.7) and save the results under the names “ASmu\_cle\_fir” and “ASmu\_con\_fir”. Then we compute the test statistics defined in equation (7.12) and carry out the permutation test.

**Code chunk 4** contains the codes related to Section 7.2.2, where the inference is under Ex-chi square distribution. This code chunk is divided into two sub-chunks.

1. In chunk 4.1, we perform simulated annealing to estimate the MLE for the cleft subjects. The results are saved under the name “cle\_fir\_SA”. Then we check the trace plots and compute the direct asymmetry measure defined in equation (7.7). The result is saved with the name “ASmu\_cle\_fir\_Exchisq”. The expectation, variance and log-likelihood corresponding to the random variable AS are computed as well.
2. In chunk 4.2, we perform simulated annealing to estimate the MLE for the control subjects. The results are saved under the name “con\_fir\_SA”. Then we check the trace plots and compute the direct asymmetry measure defined in equation (7.7). The result is saved with the name “ASmu\_con\_fir\_Exchisq”. The expectation, variance and log-likelihood corresponding to the random variable AS are computed as well.

**Code chunk 5** contains the codes related to Section 7.2.2, where the inference is under ex-Gaussian distribution. This code chunk is divided into three sub-chunks.

1. In chunk 5.1, the codes are related to evaluate the density function of the ex-Gaussian distribution given in equation (5.4). The name of this function is “exGaussian\_pdf”.
2. In chunk 5.2, we perform simulated annealing to estimate the MLE for the cleft subjects. The results are saved under the name “cle\_fir\_SA\_ExGau”. Then we check the trace plots and compute the direct asymmetry measure defined in equation (7.7). The result is saved with the name “ASmu\_cle\_fir\_ExGau”. The expectation, variance and log-likelihood corresponding to the random variable AS are computed as well.
3. In chunk 5.3, we perform simulated annealing to estimate the MLE for the control subjects. The results are saved under the name “con\_fir\_SA\_ExGau”. Then we check the trace plots and compute the direct asymmetry measure defined in equation (7.7). The result is saved with the name “ASmu\_con\_fir\_ExGau”. The expectation, variance and log-likelihood corresponding to the random variable AS are computed as well.